How a Metacognitive Strategy Helps Students Solve Mathematical Word Problems

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ABSTRACT

The present study examined how a metacognitive strategy known as self-explanation helps elementary school children solve word problems. In a series of our experiments, fifth and sixth graders were assigned to either a self-explanation group or a control group. Students in each group performed mathematical word problem tests and a transfer test. The results showed that both fifth and sixth graders in the self-explanation group outperformed those in the control group on both mathematical word problem tests and the transfer test. In addition, high self-explainers who generated more self-explanations relating to deep understanding of worked-out examples outperformed low self-explainers on both mathematical word problem tests and the transfer test. The self-explanation effect is discussed.

Key words: elementary school children, mathematics word problem solving, self-explanation, worked-out examples

1. The Aim

Many researchers have focused on the effects of metacognitive strategies in academic domains. Metacognitive strategies mean that students apply reflective thinking to problem solving or memorization tasks. It is well known that there are a variety of metacognitive strategies, for example, self-explaining, self-questioning, asking questions, answering questions, summarizing, note-taking, and drawing. Recent research has shown that self-explaining is an effective metacognitive strategy across a wide range of academic task domains (e.g., Aleven & Koedinger, 2002; Chi, 2000; Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Chi, de Leeuw, Cniu, & LaVancher, 1994; Renkl, 2002; Tajika, Nakatsu, Nozaki, Neumann, & Maruno, 2007). Students generally improve their performance better when they explain tasks such as expository texts and physics problems to themselves (Bielaczyc, Pirolli, & Brown, 1995; Chi et al., 1989; Renkl, 1997) or when they self-explain

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their own problem-solving steps (Berardi-Coletta, Buyer, Dominowsky, & Rellinger, 1995; Neuman & Schwarz, 1998, 2000; Tajika et al., 2007). Mathematics education is especially improved by metacognitive strategies that get students to learn with greater understanding.

The purpose of the present study was to examine how a metacognitive strategy known as self-explanation helps word-problem solving in elementary school children. Our research explores the effectiveness of self-explanation as a metacognitive strategy in mathematics education for elementary school children. The children in our study were fifth and sixth graders in Japanese elementary schools.

2. Studies of Self-Explanations

2-1. Studies of Self-Explanations in a Mathematics Domain

Self-explanation is one of the well-established helpful strategies for facilitating mathematical word problem solving. According to Chi (2000), self-explanation refers to utterances in which students explain the contents during learning to themselves. Self-explanation was postulated as a potential learning activity in trying to understand how students are able to learn successfully from text materials that are incomplete or to learn procedural skill from worked-out examples (Chi et al., 1989). Learning materials often include information gaps or omissions both in the text passages as well as in descriptions of the steps involved in worked-out examples. Self-explaining is now known as an effective metacognitive strategy that helps students develop deep understanding of complex academic tasks. Researchers have established the benefits of self-explaining across many academic domains for a range of ages and learning contexts. We first provide evidence for the effect of self-explaining by reviewing research on self-explaining mechanics problems and mathematical word problems.

Chi et al. (1989) analyzed the self-explanation which university students generated while studying worked-out examples and solving mechanics problems. They divided students into two groups, high self-explainers and low self-explainers, based on problem solving performance. High self-explainers solved more problems than low self-explainers. They showed that high self-explainers and low self-explainers differed with respect to both quantitative and qualitative aspects of self-explanations. High self-explainers tended to generate a greater number of self-explanations while studying worked-out examples of mechanics problems. They also tended to utter more accurate self-monitoring statements while studying worked-out examples. Chi et al. (1989) found that high self-explainers learned with understanding. They conclude that high self-explainers have specific goals when they refer back to the worked-out examples, such as looking for a method to find the value of a particular force. In contrast, low self-explainers, who initially spend less time studying the worked-out examples, have general goals that require them to reread large portions of the entire problem.

Research on self-explanation has shown facilitative effects in a domain of

mathematical problem solving (e.g., Aleven & Koedinger, 2002; Mwangi & Sweller, 1998; Nathan, Mertz, & Ryan, 1994; Neuman & Schwarz, 2000; Tajika et al., 2007). For example, Nathan et al. (1994) examined how the self-explaining process related to learning and subsequent problem-solving performance. They manipulated two kinds of problem-solving task (algebra manipulation tasks versus algebra story problem translation tasks) and two kinds of cognitive load (a high load versus a low load). University students were either prompted or not to self-explain while they generated their own solutions (high load condition) or studied worked-out example solutions (low load condition). They found that self-explaining facilitated test performance in the low load group for the story problem translation tasks but offered only a marginal advantage for the algebra manipulation tasks. Moreover, Aleven and Koedinger (2002) used geometry problems to compare selfexplanations emphasizing computer-based instructional environments to instructional methods that did not emphasize self-explanations. Students were trained to self-explain their solution steps for geometry problems within computer-based instructional environments. Aleven and Koedinger (2002) found that 10th-grade students who self-explained their solution steps during problem-solving practice within computer-based environments learned with greater understanding compared to students who did not explain their solution steps.

The function of self-explanation is to actively make sense of the presented learning materials (Chi, 2000). Self-explanation seems to be a constructive activity that engages students in active learning. Self-explanation involves several cognitive and metacognitive processes, which include generating inferences to make sense of uncertain statements relating the problems' surface features to structural features, integrating new information with prior knowledge, and monitoring what the statements refer to (Chi, 2000; Roy & Chi, 2005). As a result, the activity of self-explaining may be cognitively demanding. There is consistent evidence that even university students actually have difficulty engaging in generating a sustained level of high quality self-explanations (Atkinson, Derry, Renkl, & Wortham, 2000). When elementary school children are urged to self-explain each step of worked-out examples of mathematics word problems, they do not spontaneously self-explain their steps.

2-2. Self-Explanations Using Elementary School Children

We examined the effect of self-explanation using fifth- and sixth-graders in elementary schools in two experiments. Students were divided into two groups at each grade, the self-explanation group and the control group, respectively. Only sixth graders participated in Experiment 1 conducted by Tajika et al. (2007) and solved two kinds of ratio word problems. Fifth- and sixth-graders participated in Experiment 2 and solved different types of mathematical word problems than in Experiment 1. Mathematical word problems used in Experiment 2 consisted of four word problems including elimination and four word problems including decimals.

In Experiment 1, the participants were 53 sixth-grade children (mean age was 12 years 6 months) in an elementary school in Japan. Twenty-seven students were assigned to the self-explanation group and twenty-six students were assigned to the control group. They had studied ratios in an arithmetic class when they were fifth graders. Teachers simultaneously gave their ratio lesson to all students in their classroom. In Experiment 2, the participants were 52 fifth graders (mean age was 11 years 5 months) and 48 sixth graders (mean age was 12 years 4 months) in another elementary school in Japan. Twenty-four fifth graders were assigned to the self-explanation group and twenty-eight fifth graders were assigned to the control group. Twenty-three sixth graders were assigned to the self-explanation group and twenty-five sixth graders were assigned to the control group. All of the students had also taken an arithmetic class for the present word problems when they were fourth graders.

Both experiments had four sessions and were carried out in groups. In the first session, students took pretests and each of them took 20 minutes. Each pretest consisted of 4 word problems corresponding to a word problem test which was given after a worked-out example had been studied. In the second session, students in the self-explanation group studied two kinds of worked-out examples, an easy example and a difficult one. Students in the control group studied each of the same word problems including one solution step and its answer. In the third session, students in each group took word problem tests that consisted of each of the word problems with a time limit of 40 minutes. These word problem tests corresponded to those used in the second session. In the fourth session, one month after each word problem test, each student took a transfer test that took 40 minutes. The transfer test consisted of an 18-item word problem test, adapted from a multiplechoice test used by Mayer, Tajika, and Stanley (1991). It had three kinds of questions. One kind of question was to make a number sentence from a sentence such as, "Taro has 5 more apples than Hanako." Another kind of question was to write down the numbers to be needed to solve such a problem as, "Masao had 500 yen for lunch. He bought a sandwich for 290 yen, an apple for 70 yen, and a milk for 110 yen. How much money did he spend?" The other kind of question was to write down the operations to be carried out to solve such a problem as, "If it costs 100 yen per hour to rent roller skates, what is the cost of using the skates from 1:00 p.m. to 3:00 p.m.?" Students were asked to generate an answer to each question instead of being given a multiple-choice test. All materials were presented in Japanese.

Now, worked-out examples were shown, which were used in both experiments. Students in the self-explanation group received worked-out examples involving solution steps of each word problem and were trained with these worked-out examples. Students in the control group studied worked-out examples of the same mathematical word problems as those in the self-explanation group as the training tasks, but such worked-out examples of the word problems contained only numerical expressions and the answers. Teachers provided instruction for the word problems based on a usual method. After that, students

in the control group were told about how to solve the word problems and were instructed to understand each numerical expression as a solution step towards their answers.

Worked-out examples used in Experiment 1 were ratio word problems which contained two kinds of problems, an easy word problem and a difficult word problem. Each worked-out example contained several solution steps and its answer.

The easy worked-out example problem was as follows. "The science club has a capacity of 30 students at the elementary school. The ratio of students who hope to become members of the science club is 0.6. What is the number of students who hope to become members of the science club at the school?" The easy worked-out example problem had five solution steps and the answer in the self-explanation group (see Tajika et al., 2007).

The difficult worked-out example problem was as follows. "When the tank is filled up with water, it takes 10 minutes for the A faucet to fill up the tank and it takes 15 minutes for the B faucet to fill up the tank. When both A and B faucets are turned on at the same time, how long does it take to fill up the tank with water?" The difficult worked-out example problem had seven solution steps and the answer in the self-explanation (see Tajika et al., 2007).

Worked-out examples used in Experiment 2 were two types of word problems, which included elimination and decimals. Both types of worked-out examples contained two kinds of word problems, an easy word problem and a difficult word problem.

The easy worked-out example of the word problems including elimination was as follows. "When you buy one entrance ticket and six vehicle tickets in an amusement park, you pay 1700 yen. When you buy one entrance ticket and five vehicle tickets in the amusement park, you pay 1500 yen. How much do you pay for one vehicle ticket?" The easy worked-out example problem had six solution steps and the answer in the self-explanation group. (Step 1) You must answer the cost of one vehicle ticket. (Step 2) When you buy one entrance ticket and six vehicle tickets in an amusement park, you pay 1700 yen. (Step 3) When you buy one entrance ticket and five vehicle tickets in the amusement park, you pay 1500 yen. (Step 4) The relation between these explanations can be expressed with the diagram. (Step 5) As the diagram expresses, the difference between these two lines means one vehicle ticket. (Step 6) You can calculate the cost of one vehicle ticket as 1700-1500. (Answer) The answer is 200 yen (1700-1500=200).

The easy worked-out example of the word problems including decimals was as follows. "Yoshiko makes rubber bands to use in the class for handicrafts. The rubber bands are made from cutting a rubber string with 3.4 meters in 0.4 meters each. How many rubber bands do you have?" The easy worked-out example problem had five solution steps and the answer in the self-explanation group. (Step 1) You must answer the number of rubber bands. (Step 2) You have a rubber string with 3.4 meters. (Step 3) You divide a rubber string with 3.4 meters into 0.4 meters. (Step 4) The relation between these explanations can be expressed with the diagram. (Step 5) You can calculate the number of rubber bands as $3.6 \div 0.4$. (Answer) The answer is $9(3.6 \div 0.4 = 9)$.

The difficult worked-out example of the word problems including elimination was as follows. "When you get to the roller coaster in an amusement park, one adult costs twice as much money as one child costs. Two adults and three children cost 2100 yen. How much does each of one adult and one child cost?" The difficult worked-out example problem had eight solution steps and the answer in the self-explanation group. (Step 1) You must find the cost of one adult and the cost of one child. (Step 2) One adult costs twice as much money as one child costs. (Step 3) Two adults and three children cost 2100 yen. (Step 4) As the cost of one adult is equal to that of two children, the cost of one adult is exchanged for that of two children. (Step 5) When the relation between these explanations can be expresses with the diagram. (Step 6) Two thousands and one hundred yen mean the cost of seven children $(2 \times 2 + 3 = 7)$. (Step 7) One child costs 300 yen $(2100 \div 7=300)$. (Step 8) You can answer the cost of one adult as 300×2 . (Answer) The answer is as follows: One adult costs 600 yen $(300 \times 2=600)$ and one child costs 300 yen.

The difficult worked-out example of the word problems including decimals was as follows. "You have 1.6 liters of honey. Honey weighs 2.4 kilograms. How much does 3 liters of honey weigh?" The difficult worked-out example had seven solution steps and the answer in the self-explanation group. (Step 1) You must find the weight of 3 liters of honey. (Step 2) 1.6 liters of honey weighs 2.4 kilograms. (Step 3) The relation between these explanations can be expressed with a diagram (see Fig. 4). (Step 4) To find the weight of 3 liters of honey, you must calculate the weight of one liter of honey. (Step 5) When you divide 2.4 liters by 1.6 liters $(2.4 \div 1.6 = 1.5)$, you find the weight of one liter of honey is 1.5 kilograms. (Step 6) The relation between these explanations can be expressed with the diagram. (Step 7) The weight of 3 liters of honey is calculated by 1.5 x 3. (Answer) The answer is as follows: The weight of 3 liters of honey is 4.5 kilograms $(1.5 \times 3 = 4.5)$.

Fifteen mathematics education majors and two elementary school teachers classified each worked-out example into solution steps in Experiment 1. Two elementary school teachers and one of our members classified each worked-out example into solution steps which children could understand and explain easily in Experiment 2.

Before students wrote down their explanations in the blank spaces on the example sheet during self-explaining for each solution step, they were asked about whether they understood a problem solution at each step in both experiments. When students understood it, they answered "yes" and then wrote down their explanations. When students did not understand the problem solution at each step, they circled the "no" answer in pencil. Then, they were encouraged to write down their explanations about why they did not understand it.

The results of both experiments were as follows (see Tables 1 and 2). (1) Sixth graders in the self-explanation group of Experiment 1 outperformed those in the control group on the scores of the ratio word problem test. Fifth graders in the self-explanation group of Experiment 2 outperformed those in the control group on the total scores of both word problem tests. (2) Sixth graders in the self-explanation group of Experiment 1 also

outperformed students in the control group on the scores of the transfer test. However, fifth and sixth graders in the self-explanation group of Experiment 2 were similar to those in the control group on scores of the transfer test.

Table 1 Mean Scores (Ms) and Standard Deviations (SDs) for Each Group as a Function of Test Type in Experiment 1

Group	Ratio Word Problem Test		Transfer Test	
	\mathbf{M}	SD	M	SD
Self-Explanation (n=27)	12.00	3.87	15.37	2.68
Control (n=26)	8.12	2.63	12.81	3.73

Table 2 Mean Scores (Ms) and Standard Deviations (SDs) for Each Group as a Function of Test Type in Experiment 2

Group	Ratio Word Problem Test		Transf	Transfer Test	
	M	SD	M	SD	
		Sixth Grade	rs		
Self-Explanation (n=23)	12.04	3.66	9.73	3.67	
Control (n=25)	12.64	3.15	8.24	4.54	
		Fifth Grader	<u>'S</u>		
Self-Explanation (n=24)	10.04	4.33	9.79	4.21	
Control (n=28)	7.68	3.86	9.64	3.81	

Students in the self-explanation group were analyzed on their statements of solution steps in more detail. Students in the self-explanation group were divided into two groups, self-explainers of high quality (high self-explainers) and self-explainers of low quality (low self-explainers). We use the term high self-explainers to refer to students who demonstrate the generation of inferences and use more frequent metacognitive monitoring to understand statements and low self-explainers to refer to students who generate paraphrases and rereading statements. High self-explainers would generate more self-explanations relating to deep understanding. High self-explainers generated more fine-grained self-explanation inferences at each solution step on word problems. When low self-explainers were required to self-explain each solution step of especially the difficult ratio word problem, they often said that they did not know what to explain. So, we hypothesized that high self-explainers would solve more word problem and transfer tests

than low self-explainers when quality of self-explanation influences students' problem solving processes.

When students in each experiment were divided into high self-explainers and low self-explainers, fifteen students were classified as high self-explainers and twelve students were classified as low self-explainers in Experiment 1. In Experiment 2, eleven sixth graders and eight fifth graders were classified as high self-explainers. Twelve sixth graders and sixteen fifth graders were classified as low self-explainers.

The results showed that high self-explainers outperformed low self-explainers on both word problem and transfer tests in both experiments (see Tables 3 and 4).

Table 3 Mean Scores (Ms) and Standard Deviations (SDs) for High Self-Explainers and Low Self-Explainers as a Function of Test Type in Experiment 1

Group	Ratio Word Problem Test		Transfer Test	
	M	SD	M	SD
High Self-Explainers (n=15)	14.47	2.75	16.60	1.31
Low Self-Explainers (n=12)	8.91	2.43	13.83	3.02

Table 4 Mean Scores (Ms) and Standard Deviations (SDs) for High Self-Explainers and Low Self-Explainers as a Function of Test Type in Experiment 2

Group	Ratio Word Problem Test		Transf	Transfer Test	
	M	SD	M	SD	
		Sixth Grader	<u>'s</u>		
High Self-Explainers (n=11)	13.67	3.11	10.45	4.06	
Low Self-Explainers (n=12)	10.54	3.48	8.85	3.29	
		<u>Fifth Grade</u>			
High Self-Explainers (n=8)	13.75	1.91	13.25	2.05	
Low Self-Explainers (n=16)	8.19	3.85	8.06	3.75	

3. Discussion

It is well known that the use of worked-out examples has proved effective in improving performance in a variety of domains (Atkinson et al., 2000). The results from our first experiment and from fifth grade students in the second experiment suggest that when students first respond to the solution steps of each worked-out example with yes or no and then self-explain them, the use of worked-out examples are effective with respect to

solving word problems having similar structure to worked-out examples. However, sixth graders in the self-explanation group solved as many word problems as those in the control group in Experiment 2. The reason why similar results have been obtained in sixth graders in Experiment 2 seems to be that word problems including elimination and decimals are easy to solve. When we compare the scores of ratio word problems of the control group with those of word problems including elimination and decimals of the control group, we find that the scores of the ratio word problems are much lower than those of the word problems including elimination and decimals. So, self-explaining worked-out examples was not effective.

The results also showed that while solution steps of the worked-out examples might have helped students solve the transfer test in Experiment 1, they might not have helped students solve the transfer test in Experiment 2. Though there was no difference in performance on the transfer test between the self-explanation group and the control group of fifth graders in Experiment 2, the results showed that high self-explainers outperformed low self-explainers. These results seem to provide support for self-explaining. When students self-explain worked-out examples of one type of word problem, they might solve another type of word problem better.

The result involving high self-explainers and low self-explainers supported the hypothesis that high self-explainers who generate more self-explanations relating to deep understanding of word problems would outperform low self-explainers on word problem and transfer tests. High self-explainers generated more fine-grained self-explanation inferences at each solution step on word problems. For example, a student self-explained a solution step of a difficult worked-out example as follows: It takes 10 minutes for the A faucet to fill up the tank. So, one minute means 1/10 of 10 minutes. The ratio at which the tank is filled up with water is 1/10.

As stated earlier, we have pointed out that monitoring usually refers to both comprehension and a comprehension failure. Even high self-explainers sometimes self-explain solution steps as a comprehension failure type of monitoring. However, monitoring activities by high self-explainers were different from those by low self-explainers. High self-explainers tried to find the flaw in their knowledge that caused the comprehension failure and tended to monitor the comprehension failure more clearly. Even if some students gave yes responses to solution steps in the worked-out examples, they only repeated sentences of solution steps, instead of explaining solution steps to themselves. In Experiment 1, fifteen high self-explainers generated more self-explanations and more fine-grained explanations relating to deep understanding of ratio word problems than twelve low self-explainers. In Experiment 2, eleven high self-explainers of the sixth graders and eight high self-explainers of the fifth graders generated more self-explanations and more fine-grained explanations. Students self-explained solution steps of word problems to actively integrate prior knowledge with information contained in solution steps. Findings indicate that while high self-explainers used self-explanations as a metacognitive strategy to make

incomplete understanding complete, low self-explainers did not learn much from solution steps in the worked-out examples during self-explaining. Judging from the results of high self-explainers, it may be suggested that self-explanation is an effective metacognitive strategy when dealing with not only mathematical word problems but also transfer problems. When metacognitive strategies such as self-explanations are effectively used to learn from worked-out examples, students may improve mathematical solution skills by reconstructing incomplete mental models they have.

The low self-explainers gave some no responses, generated fewer explanations, and often repeated sentences of solution steps of the difficult word problem. Even though they answered "no", they had to explain what they could not understand. When they gave no responses to solution steps, they were still required to self-explain their answers. Many fifth graders in the self-explanation group found it difficult to explain why they did not know. When the low self-explainers were required to self-explain each solution step of the difficult word problem, they often said that they did not know what to explain. Even some sixth graders in both experiments gave no responses to solution steps in the difficult worked-out example and repeated sentences of solution steps and wrote them down on a sheet of the worked-out example. Low self-explainers sometimes said that they did not understand what the sentences meant and often monitored the solution steps in which they failed to understand the meaning of the sentences.

4. Conclusion

Research on self-explanations suggests that generating self-explanations is useful in general. Chi (2000) also points out that generating incorrect self-explanations does not depress effective performance. When they first gave yes or no responses to the questions about whether they understood a problem solution at each step and then self-explained each worked-out example in the present experiments, we attempted to make students self-explain more. Self-explaining may have encouraged students to integrate information presented in word problems with their prior logico-mathematical knowledge.

The most commonly used technique in mathematics textbooks for helping students solve word problems is to provide worked-out examples. Worked-out examples typically present solutions in a step-by-step fashion. Worked-out examples have three steps. They consist of a problem formulation, solution steps, and the final solution (answer). We found that worked-out examples became a more effective technique when even elementary school children self-explained each solution step of worked-out examples.

Although research on mathematical problem solving by self-explaining worked-out examples has found facilitative effects using this instructional approach, two main factors influencing performance can be stated on the basis of the above results using fifth and sixth graders (see Renkl, 2005). (1) The importance of providing elaborated solution steps in worked-out examples. Few elementary school children generate inferences to explain each solution step in worked-out examples. Many of them only reread each solution step

or repeat statements. We should ask students to self-explain each solution step in worked-out examples. As a result, we should provide elaborate solution steps in worked-out examples which students easily generate inferences to explain. (2) Traditional worked-out examples show just one type of solution step procedure. However, elementary school teachers often provide students multiple solution procedures dependent on mathematical problem types. We need to provide several types of solution step procedures in a worked-out example.

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